Maxwell distribution and its use in evaluating molecular velocities (average, root mean square and most probable) and average kinetic energy.

Content Type	
Status	Done
Chapter	Gaseous State
Course	Graduation
Reference	
Subject	Physical Chemistry
Topic	
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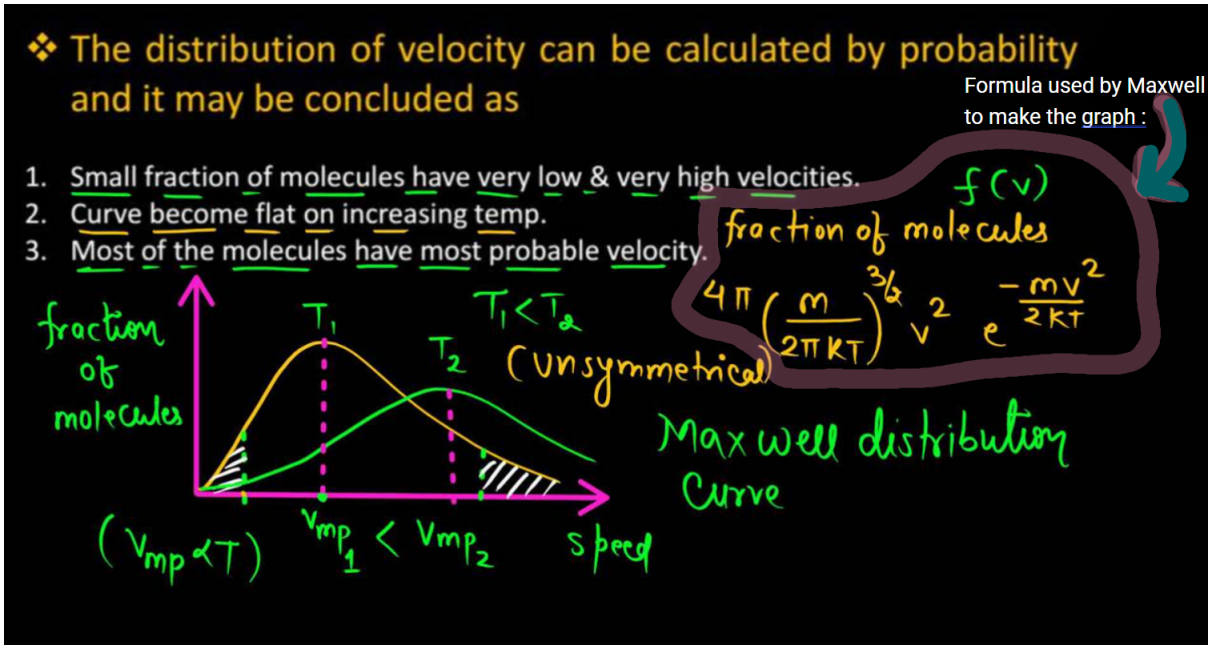
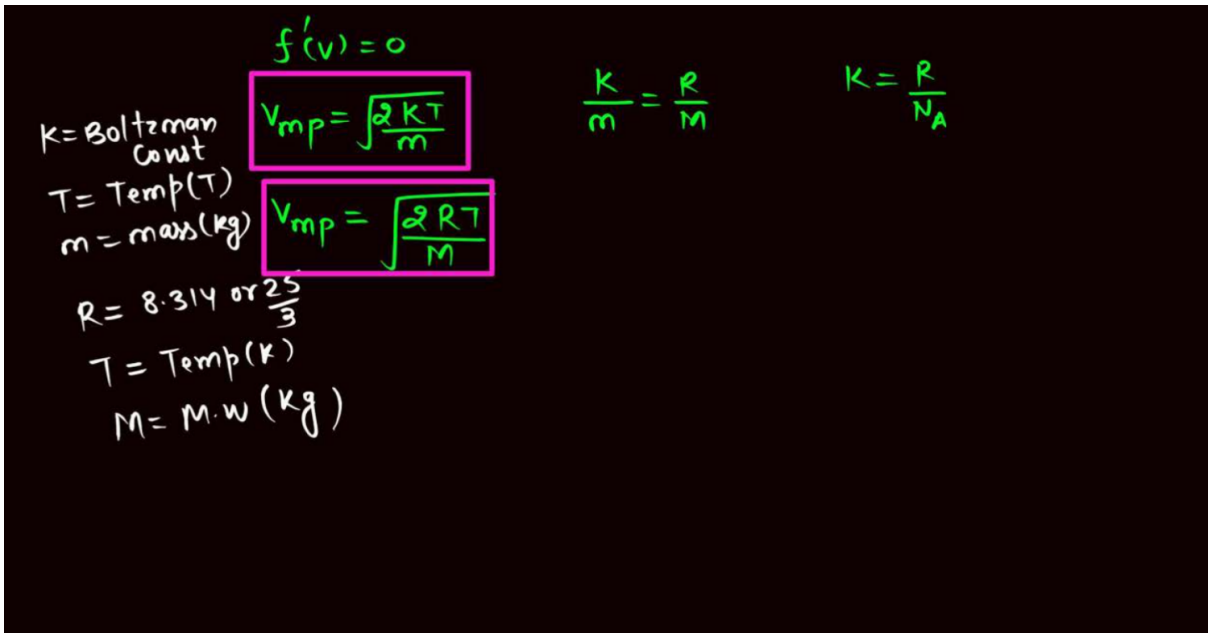
# Progress (%)	100
Difficulty Level	Medium
Mistake type	
Previous Year Questions	

Maxwell Distribution

The Maxwell–Boltzmann distribution is a probability distribution that describes the speed of particles in a gas. It states that, among a collection of particles of the same temperature, the distribution of particle speeds follows a particular form determined by the temperature and mass of the particles.


The distribution of velocity can be calculated by probability and it may be concluded as
1. Small fraction of molecules have very low & very high velocities.
2. Curve become flat on increasing temp.
3. Most of the molecules have most probable velocity.
Formula used by Maxwell to make the graph :
f(v)
fraction of molecules
$4\pi\left(\frac{m}{2\pi KT}\right)^{3/2} v^2 e^{\frac{-mv^2}{2KT}}$
fraction of molecules
T_1
T_2
$T_1 < T_2$
(unsymmetrical)
Maxwell distribution Curve
$(V_{mp} \propto T)$
$V_{mp_1} < V_{mp_2}$
Speed

Vmp=Most probable velocity whch is directly proportional to Temperaature.



This distribution is used to evaluate molecular velocities (average, root mean square and most probable) and average kinetic energy.

Molecular Velocities

According to Maxwell distribution, the velocity of individual molecules in a gas follows a normal distribution, with the mean velocity being the root mean square (RMS) velocity and the most probable velocity being the peak of the curve.

Average Speed

The average speed (u) of a particle is given by the equation:

u = (8 kT/π m)^1/2

where:

- k is the Boltzmann constant
- T is the temperature(K)
- m is the mass of particles

Root Mean Square (RMS) Speed

The root mean square (RMS) speed (u_rms) of a particle is given by the equation:

u_rms = (3kT/m)^1/2

where:

- k is the Boltzmann constant
- T is the temperature(K)
- m is the mass of particles

Most Probable Speed

The most probable speed (u_mp) of a particle is given by the equation:

u_mp = (2kT/m)^1/2

where:

- k is the Boltzmann constant
- T is the temperature(K)
- m is the mass of particles

Average Kinetic Energy

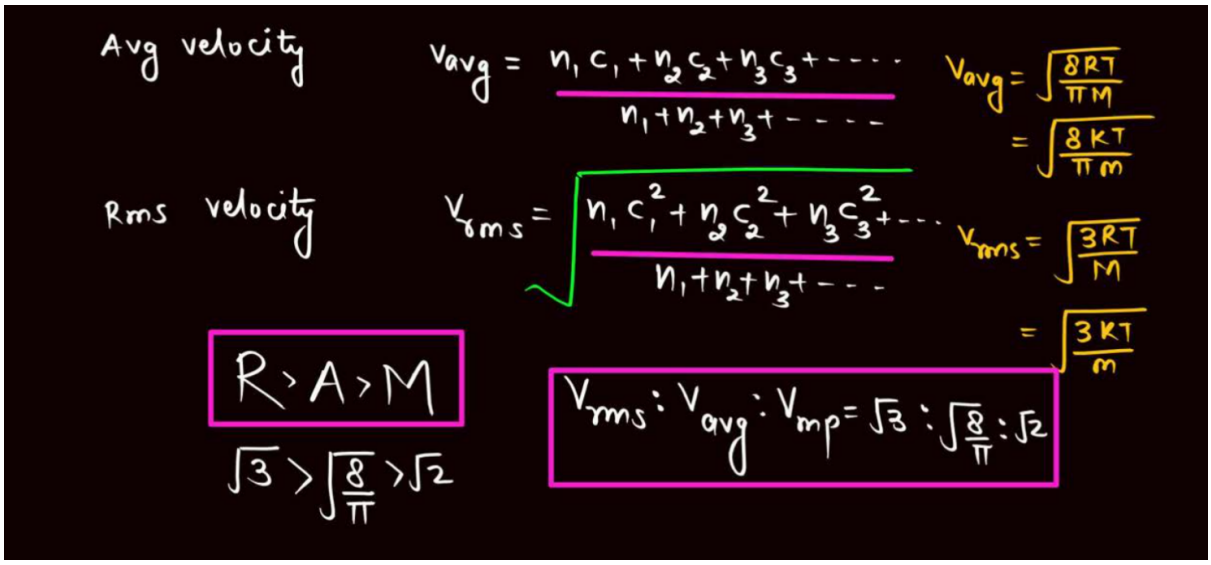
The average kinetic energy (E_avg) of a particle is given by the equation:

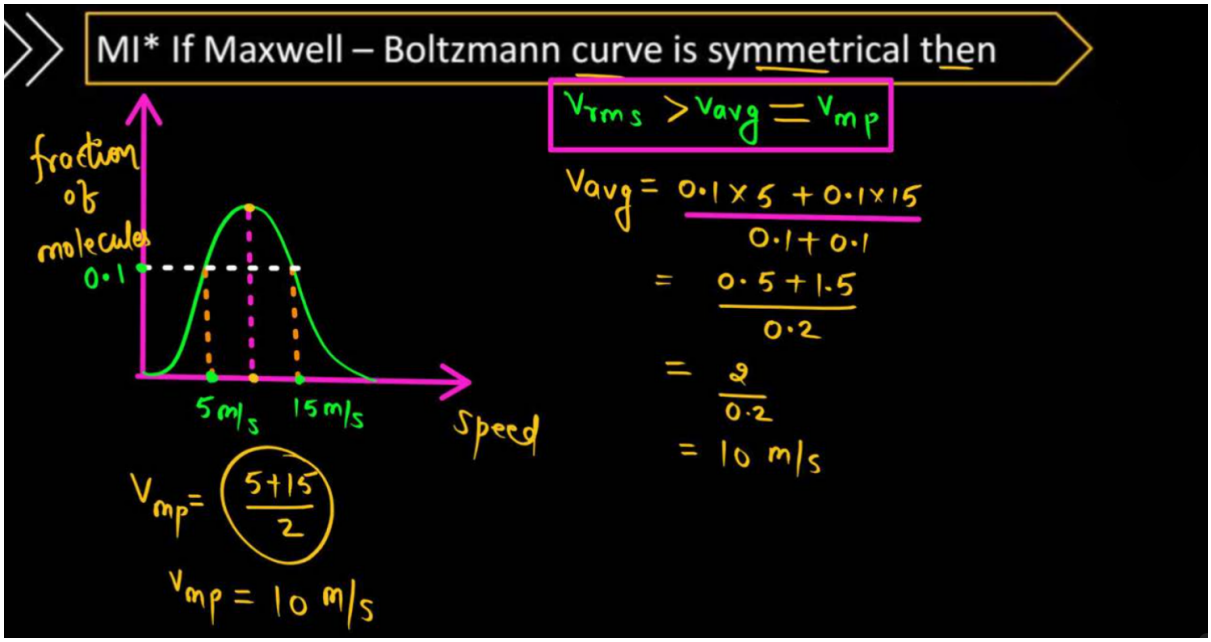
E_avg = (3/2)kT

where:

- k is the Boltzmann constant
- T is the temperature(K)

Relation between Molecular Velocities


Avg velocity
$v_{avg} = \frac{n_1c_1 + n_2c_2 + n_3c_3 + \cdots}{n_1 + n_2 + n_3 + \cdots}$
$v_{avg} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8KT}{\pi m}}$
Rms velocity
$v_{rms} = \sqrt{\frac{n_1c_1^2 + n_2c_2^2 + n_3c_3^2 + \cdots}{n_1 + n_2 + n_3 + \cdots}}$
$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3KT}{m}}$
R > A > M
$\sqrt{3} > \sqrt{\frac{8}{\pi}} > \sqrt{2}$
$v_{rms} : v_{avg} : v_{mp} = \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2}$



$$V_{rms} = \sqrt{\frac{0.1 \times 5^2 + 0.1 \times 15^2}{0.1 + 0.1}}$$

$$= \sqrt{\frac{2.5 + 22.5}{0.2}}$$

$$= \sqrt{\frac{25}{0.2}}$$

$$= \sqrt{125}$$

Q&A

Q. Four particles have speed 2, 3, 4 & 5 cm/s respectively find Vrms ?

$$V_{rms} = \sqrt{\frac{1 \times 2^2 + 1 \times 3^2 + 1 \times 4^2 + 1 \times 5^2}{1+1+1+1}}$$

$$= \sqrt{\frac{4+9+16+25}{4}} \text{ cm/s}$$

$$= \sqrt{13.5} \text{ cm/s}$$

Q. The most probable speed of 8 gm H_2 is 200m/s find avg K.E of H_2 gas?

$V_{mp} = \sqrt{\frac{2RT}{M}}$ $\qquad$ $KE = \frac{3}{2}nRT$

$$(200)^2 = \frac{2 \times RT}{2 \times 10^{-3}}$$

$$(RT) = 4 \times 10^4 \times 10^{-3}$$

$$= 40$$

$$= \frac{3}{2} \times \left(\frac{8}{2}\right)^2 \times 40$$

$$= 240$$

Q. The density of gas filled electric lamp is 0.75 kg/m^3. After the lamp has been switches on, the pressure in it increases from 4×10^4 Pa to 9×10^4 Pa find the increase in V_{rms}

$$V_{rms} = \sqrt{\frac{3RT}{M}} \qquad d = \frac{PM}{RT}$$

$$= \sqrt{\frac{3P}{d}} \qquad \frac{RT}{M} = \frac{P}{d}$$

$$\Delta V_{rms} = \sqrt{\frac{3\Delta P}{d}} = \sqrt{\frac{3(5\times10^4)}{0.75}} = \sqrt{\frac{15\times10^6}{75}} = \sqrt{\frac{10^6}{5}} \text{ m/s}$$

Q. If V_{rms} of gas is $30\sqrt{R}$ m/s at 27°C then find molar mass of gas ?

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$30\sqrt{R} = \sqrt{\frac{3RT}{M}}$$

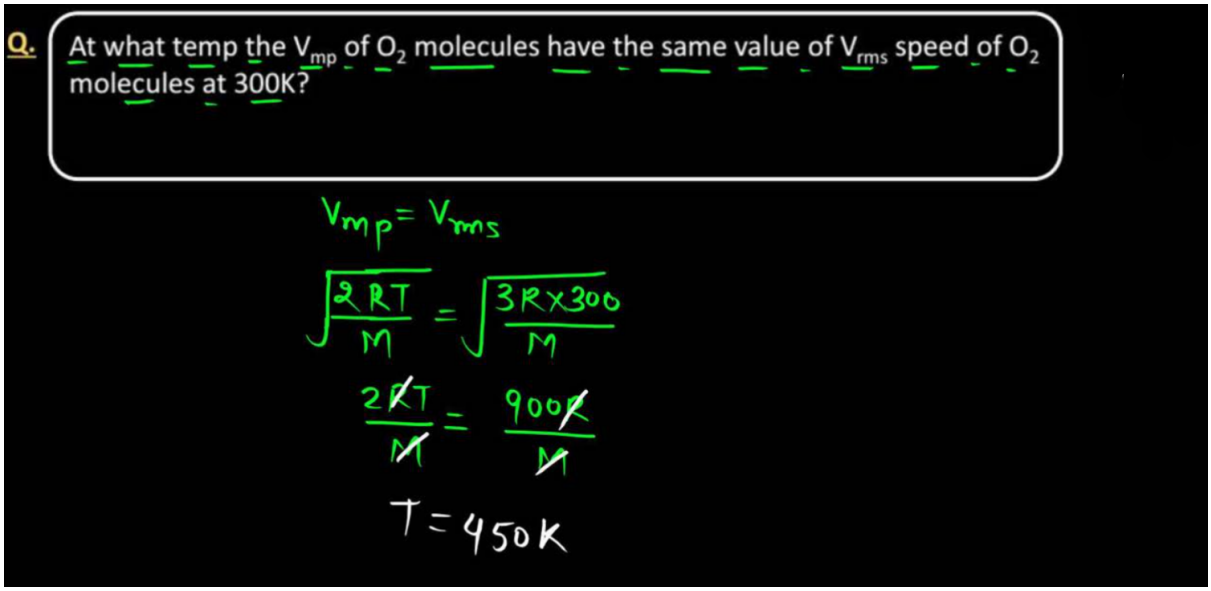
$$900R = \frac{3R \times 300}{M}$$

$$M = 1 \text{ kg}$$

Q. Calculate V_{rms} of O_3 kept in a closed container at 20°C and 82 cm Hg pressure?

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{\frac{3 \times \frac{25}{3} \times 293}{48\times10^{-3}}} \text{ m/s}$$


Q. At what temp the V_{mp} of O_2 molecules have the same value of V_{rms} speed of O_2 molecules at 300K?
$V_{mp} = V_{rms}$
$\sqrt{\frac{2RT}{M}} = \sqrt{\frac{3R \times 300}{M}}$
$\frac{2RT}{M} = \frac{900R}{M}$
$T = 450K$